DOCUMENT RESUME

ED 451 036 SE 064 548

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TITLE Brousseau in Action: Didactical Situation for Learning How To Graph Functions.

PUB DATE 1999-00-00

NOTE 9p.

PUB TYPE Guides - Non-Classroom (055)

EDRS PRICE MF01/PC01 Plus Postage.

DESCRIPTORS Educational Technology; *Functions (Mathematics); Graphing Calculators; *Graphs; High Schools; Higher Education; Mathematics Activities; Mathematics Instruction; Teaching Methods

ABSTRACT

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Brousseau in action: Didactical situation for learning how to graph functions

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Abstract

In this paper we present a proposal for teaching how to graph functions on the basis of the teaching theory of didactical situations implemented by the French author Guy Brousseau. The proposal is aimed at students in 12th grade in highschool (USA) or in a freshman calculus course. We use the TI-92 graphic calculator as a teaching aid.

Introduction

The graph of a function, in most of our curricula, is taught in the last courses of Bachillerato (levels 10-12 in the USA) or in the first calculus course at college. For instance, at the Colegio de Ciencias y Humanidades (CCH), one of the two bachillerato subsystems at the Universidad Nacional Autónoma de México, the topic is included in a unit called Applications of derivative, in the first course of calculus (at level 12).

All the calculus texts that we use in teaching are written from a traditional point of view. That is, they present all the concepts and theory involved, give some examples and applications, and then ask the students to solve exercises and problems. We believe that most of our math teachers teach the mathematics concepts in the same way they are presented in the text they are using. Nevertheless, this traditional way of teaching contradicts the three goals of our

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Institution: Learn to learn; learn to do, and learn to be. In the CCH math curriculum, reformed in 1996, we adopt the constructivist philosophy of teaching and problem solving. We, as mathematics teachers, are ourselves constructing the appropriate knowledge to teach under such philosophy. We are searching for a theoretical framework that allows us a better performance as teachers and educators. In this context, we are certain that the didactics proposed by the French researcher Guy Brousseau has a great educational potential. Besides, we can't abstract ourselves from the important roll the new technology is playing in the teaching and learning processes. Therefore, we are engaged in some kind of struggle against the somewhat negative attitude toward the use of calculators and computers in the classroom, held by many of our colleagues and authorities.

The teaching proposal presented in this paper is product of that search. Its main objective is present a didactical proposal for teaching function graphing based in Brousseau didactique, with the aid of a TI-92 graph calculator, and, at the same time, review the major concepts and ideas of this didactique using a concrete example.

Constructivism, problem solving, and Brousseau

Constructivism as learning theory arises from the need to account for, in a more rational way, how human beings learn. It is based on the thesis that knowledge is not transferred from one person to another, but the individual who learns constructs its own knowledge. About the act of thought, Dewey highlights five steps: a) perception of a difficulty; b) determination and definition of that difficulty; c) proposal of a possible solution; d) development of the consequences of the proposal; and e) later observations and research that lead to the acceptance o rejection of the proposal (Aebli, page 31)

Claparede, on his part, states that all actions have the intention to readapt the subject to the environment when the balance between them is broken. This break may come from the modification of the environment, but, and more often, is the individual who breaks the balance only to try to recover it in a upper level. He notes that the difficulty that leads the individual to think becomes apparent, at the beginning, in a need that becomes a **problem** or a **question**. This need gives interest to the object of the activity or to the activity itself (Aebli, pp. 31-32).

With the prior paragraphs we wish to note the importance that the constructivist theory puts on problem solving as a tool for learn mathematics. The learner's need for solve certain problematic situation gives rise to the reflection act and, in consequence, to knowledge. Therefore, in **teaching mathematics we should encourage student's problem solving as a mean to foster reflection and motivate action.**

Another key issue in our proposal is the teamwork. To that respect Piaget says that the formation of grouped operations in the student (in opposition to habit formation, which are relatively isolated conducts) is a result of her cooperation.

And cooperation among individuals is condition of the operatory organization of thought. In this sense, a good teaching involves socialized activities (Aebli, page 74). Besides, the teamwork allows standardization of knowledge among peers; fosters discussion of different solutions and strategies of solution; develops in the student the ability of communicate mathematical ideas; and also permits development of arguments that validate the statements done.

On the other hand, and in this same constructivist context, Brousseau differentiate three situations in the teaching process (adapted from Bessot):

- **Non-didactical situation:** with respect to knowledge *S*, is that situation that is not explicitly organized to allow the learning of *S*. For instance, at the secondary level, all that has to do with operation with naturals may be considered as a non-didactical situation.
- **Didactical situation;** with respect to knowledge *S*, is that situation design explicitly to encourage *S*. We can consider as didactical all the tasks done in a classroom with which the teacher intents to teach *S*, and with which the student is forced to learn *S*.
- **A-didactical situation:** with respect to knowledge *S*, is that situation that contains all the conditions that permit the student to establish a relationship with *S*, regardless of the teacher. The actions that the student does, and the answers and arguments that she produces depend on her relationship (no completely explicit) with *S*, i. e. with the "problem" that she must solve or wit the difficulty that she must overcome. In this case, a process of devolution of responsibility is in action.

In his text "Theory of didactical situations in mathematics", Guy Brousseau states the following:

> *The modern conception of teaching therefore requires the teacher to provoke the expected adaptation in her students by a judicious choice of "problems" that she puts before them. These problems ... must make the students act, speak think and evolve by their own motivation... The student knows very well that the problem was chosen to help her to acquire a new piece of knowledge, but she must also know that this knowledge is entirely justified by the internal logic of the situation and she can construct it without appealing to didactical reasoning. Not only* can *she do it, but she* must *do it because she will have truly acquire this knowledge only when she is able to put it to use by herself in situations which she will come across outside any teaching context and in the absence of any intentional direction. Such a situation is called an* adidactical situation...
>
> *This situation or problem chosen by the teacher is an essential part of the broader situation in which the teacher seeks to devolve to the student*

> *an adidactical situation which provides her with the most independent and most fruitful interaction possible... She is thus involved in a game with the system of interaction of the student with the problems she gives her. This game, or broader situation, is the* didactical situation. (pp. 30-31)

On the other hand, Brousseau tells us that the *didactical contract* is the game rules, and the strategies of the didactical situation. It is the justification that the teacher has for presenting the situation. It depends closely on the specific knowledge that she wants to teach:

> *The student interprets the situation put before she, the questions asked to her, the information given, and the constrictions imposed depending on what the teacher reproduces, aware of it or not, in a repetitive way in her teaching practice. We are especially interested in what is specific of the knowledge to teach: we call* didactical contract *to the set of specific attitudes that the student expects from her teacher, and the set of specific attitudes the teacher expects from her student.* (Brousseau, quoted by Polo)

The setting up of a didactical contract between teacher and student permits the presentation and development of didactical situations formed by adidactical situations.

In every adidactical situation is present a validation process that can be established among the students o between student and teacher. To this respect, Broussea says (Brousseau, page 12):

> *...it can happen that one student's propositions are discussed by another student, not from the point of view of the language (the message is or is not understood) but from the point of view of the validity of the content (that is to say, its truth ot its efficacy)...*
>
> *We call these spontaneous discussions about the validity of strategies "validation phases". They appear as a means of action. The students use them a means of encouraging their partners to carry out the proposed action.*

Another concept that plays a key roll in Brousseau's *didactique* is institutionalization:

> *In institutionalization, she* (the teacher) *defines the relationships that can be allowed between the student's "free" behaviour or production and the cultural or scientific knowledge and the didactical project; she provides a way of "reading" these activities and gives them a status.* (Brousseau, page. 56)

The student, in facing adidactical situations constructs the piece of knowledge tah the teachers wants to teach, but such knowledge must be in accordance with the

scientific or cultural knowledge socially accepted. Thus, the teacher must situate the student's production in this context. Institutionalization is the mean by which the teacher does such location.

On the use of calculators

The use of graphic calculators in classroom has been implemented and recommended, see for instance the NCTM Standards. But we need to know how kind of knowledge the student constructs from the use of such an artifact, and what kind of teaching activities do we need to implement to foster the piece of knowledge we want to teach. Vygotsky states that artificial systems can improve man's cognitive capacities by developing his ability to act on the environment (Dominique and Trouche). In some way this is what we are looking for with the use of technology. The authors note that if cognition evolves interacting with the environment, accommodation to artifacts may impact the cognitive development and the construction of knowledge. But this is achieved when the artifact becomes an instrument as a psychological construct. This happen when the individual interacts with the artifact and acquires knowledge that lead to a different and better use of the artifact. Thus, to use graphic calculators as an learning instrument, students must develop the notion of calculator as an instrument by means of the interaction with it. And this mere fact facilitates construction of knowledge by the student.

The use of technology (in particular, calculators and computers) in the classroom also has a motivating aspect. Sfard and Leron pointed out that, unlike the teacher, computers are obedient but obstinate, demanding but consistent and predictable. And this is what a student may consider as a fair play. He or she can interact with the artifact, get to know its working rules and feels free to experiment with it, since the machine wont negotiate with the student, and he or she feels that is dealing with something that must be taken seriously. The computer can speak to the student, responds to anything the student is doing and, in some way, they become an interlocutor with which he or she may have a conversation on problem solving. In this environment, mistakes acquire an important role, they encourage the student to seek other strategies of solution and prove new conjectures.

Brousseau in the classroom: the case of graphing functions

In accordance to what we say in the previous paragraphs, the teaching of any topic in mathematics should begin with the presentation to the students of one or several adidactical situations in which the piece of knowledge to teach is contained.

In the specific case of graphing a function, we present a series of different problems whose only common point is the graph of a function. These problems must be solved in teams of at most three students. Depending on the nature of the class, the teacher must decide if all the teams will work a problem at a time or if the different problems will be solved at the same time. The real important thing

here is that each team knows what problem the others are solving. Once the different teams got the solution (or solutions) of the problems, each team explains the strategy of solution followed and argues toward the validity of their solution. At the end, all the teams must have a "valid" solution for all the problems and the different ways in which they obtain this solution. Once the problem solving and the validation of results stages are completed, the teacher summarizes the results and gives appropriate names for the concepts used.

The problems have different levels of difficulty and they refer to different branches of knowledge. This is so because we want the student reviews and applies concepts studied previously. At this level, the student must have basic knowledge on physics, chemistry, and biology, as well as a clear understanding of functions. Therefore, besides the piece of knowledge that we want explicitly to teach, the problems foster the review of concepts in other fields of knowledge and link mathematics with such fields.

When dealing with the problems, the teacher must encourage the use of derivatives to determine critical points in order to introduce the first and second derivative criteria in finding critical points. It is useful to have the function and its derivative plots in the same coordinate plane, so the student can compare the two functions. The teacher must also encourage a discussion on behaviour of the function at infinity and singular points.

Problems

1. A farmer knows that when he starts a honeycomb with 50 bees, in a week there will be 90 bees, and with time bee population approach about 75 200 insects, before population begins to decrease (about 25 weeks).

   If we apply the population growth model

   $$P(t) = \frac{a}{1 + be^{-ct}}$$

   Where P(t) is the bee population at time t, what will the value of the parameters a, b, and c, for t between 0 and y 25 weeks, be ? In accordance with the model, in which instant will the growth be faster? Plot the population growth as function of time. Describe the plot.

2. A research team has experimentally determined that the speed of swimming of migrating fish traveling countercurrent is 50% grater than the speed of the current. They hypothesize that with this speed the fish minimizes the required energy to travel certain distance. Which energy expression as function of fish speed, current speed, and distance fulfills this hypothesis? What shape does the plot of this function have? Describe it.

3. The way in which a light ray propagates in passing from one medium to another of different density is a problem that worried the scientific world

since ancient times. Ptolemy of Alejandria, in the 2nd century B. C., tried to obtain experimentally a law of refraction of light, but he didn't succeed. The Dutch Willebrord Snell Van Royen discovered such law in 1620.

The Snell law states that the sine of the angle between the incident light ray and the normal line at the interface of the two media divided by the sine of the angle between the refracted ray and the same normal line is a constant:

$$\frac{\text{sen}ß_1}{\text{sen}ß_2} = k$$

With k the quotient of light speeds in the two media. The French scientist Pierre Fermat assumed that, in an homogeneous medium the light travels from one point to another following a path that requires the least time (Fermat principle), and with this assumption derived theoretically the Snell law.

Find the function derived by Fermat. Plot this function and describe it. How can you obtain the Snell law?

4. Find the equation of the straight line that passes through (3,7) and determines the least area in the first quadrant. Plot the graph of the area as function of slope of the line and as function of the b parameter. Plot and describe both graphs.

5. A working man wants to build a rain gutter from a metal sheet of width 30 cm by bending up one third of the sheet on each side. Which value must the bending angle have so the gutter carries the maximum amount of water? How does the volume of water depend on the bending angle? Plot and describe the graph.

Note: the problems were adapted from, Giancoli, Stewart, and Tikhomirov.

References

Aebli, H. *Una didáctica fundada en la psicología de Jean Piaget*, Kapeluz, Buenos Aires, 1958.

Bessot, A., Panorama del quadro teorico della didactica matematica in *L'Educazione Matematica*, 15(4) Vol. 1 (May), 1994.

Brousseau, G., *Theory of didactical situations in mathematics*, Kluwer Academic Publisher, 1997.

Giancoli, D. C., *Physiscs*, 4th ed. Prentice-Hall, 1995.

Guin, D., and L. Trouche, *The complex process of converting tools into mathematical instruments: the case of calculators*, in International Journal of Computers for Mathematical Learning, 3, 1999.

Polo, M, *Il contratto didattico come strumento di lettura della pratica didattica con la matematica*, in L'Educazione Matematica, 1999.

Sfard, A., and U. Leron, *Just give me a computer and I will move the earth: programming as a catalyst of a cultural revolution in the mathematics classroom,* in International Journal of Computers for Mathematical Learning, 1, 1996.

Stewart, J., *Calculus, concepts and contexts,* Brooks/Cole Publishing Company, 1998.

Tikhomirov, V. M., *Stories about Maxima and Minima*, American Mathematical Society, 1990.s